Mechanical problems of superplastic fill-forming bulge solved by one-dimensional tensile and two-dimensional free bulging constitutive equations*

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Abstract Because of the strong structural sensitivity of superplasticity, the deformation rule must be affected by stress-state. It is necessary to prove whether one-dimensional tensile constitutive equation can be directly generalized to deal with the two-dimensional mechanical problems or not. In this paper, theoretical results of fill-forming bulge have been derived from both one-dimensional tensile and two-dimensional bulging constitutive equation with variable m value. By comparing theoretical analysis and experimental results made on typical superplastic alloy Zn-wt22%Al, it is shown that one-dimensional tensile constitutive equation cannot be directly generalized to deal with two-dimensional mechanical questions. A method to correct deviation between theoretical and experimental results is also proposed.

Keywords: superplastic, fill-forming bulge, constitutive equation.

Since Holt first used Backofen's constitutive equation with constant m[1] in Ref. [2] to investigate the bulging process of a sheet forming into 90° V grooves and obtained the expression between bulging pressure and round radius by iteration method, Song et al.[3] have used superplastic tensile constitutive equation with variable m to analyze mechanically superplastic fill-forming bulge. By comparing results in Ref. [2] with those in Ref. [3], it is concluded that although the theoretical results by applying constitutive equation with variable m are much more precise than those by applying constitutive equation with constant m, there still exists much deviation between the theoretical and the experimental results. One of the main reasons is that they used one-dimensional tensile constitutive equation. By applying superplastic one-dimensional tensile and two-dimensional free bulging constitutive equation with variable m derived in Refs. [4] and [5] respectively, we made mechanical analyses on superplastic fill-forming bulge, and then compared the two results, which showed that the one-dimensional tensile constitutive equation cannot be directly generalized to deal with the problems of superplastic fill-forming bulge.

1 Theoretical basis

The theoretical analysis was based on one-dimensional tensile and two-dimensional free bulging constitutive equations with variable m proposed in Refs. [4] and [5] respectively, which can be obtained by simulating the experimental curves of $m - \lg\dot{\varepsilon}$ obtained from one-dimensional tensile and two-dimensional bulging tests with negative power function.

If the curve of $m - \lg\dot{\varepsilon}$ is symmetric, then

$$\dot{\varepsilon}_x = \dot{\varepsilon}_{\max,x} \exp\left\{\frac{\eta_x \ln 10}{\sqrt{m_{\max,x}/m_{k,x} - 1}} \cdot \tan\left(\frac{\sqrt{m_{\max,x}/m_{k,x} - 1}}{m_{\max,x}\eta_x} \lg\frac{\sigma_x}{k_x}\right)\right\}; \quad (1)$$

if the curve of $m - \lg\dot{\varepsilon}$ is unsymmetrical, it can be divided into sections, and then constitutive equation with variable m is

$$\dot{\varepsilon}_{a,x} = \dot{\varepsilon}_{\max,x} \exp\left\{\frac{\eta_{a,x} \ln 10}{\sqrt{m_{\max,x}/m_{ka,x} - 1}} \cdot \tan\left(\frac{\sqrt{m_{\max,x}/m_{ka,x} - 1}}{m_{\max,x}\eta_{a,x}} (\lg\sigma_x - \lg k_x)\right)\right\},$$
$$\lg\dot{\varepsilon}_{a,x} \leqslant \lg\dot{\varepsilon}_{\max,x},$$
$$\dot{\varepsilon}_{b,x} = \dot{\varepsilon}_{\max,x} \exp\left\{\frac{\eta_{b,x} \ln 10}{\sqrt{m_{\max,x}/m_{kb,x} - 1}} \cdot \tan\left(\frac{\sqrt{m_{\max,x}/m_{kb,x} - 1}}{m_{\max,x}\eta_{b,x}} (\lg\sigma_x - \lg k_x)\right)\right\},$$
$$\lg\dot{\varepsilon}_{b,x} \geqslant \lg\dot{\varepsilon}_{\max,x}, \quad (2)$$

where, m is the strain rate sensitivity index; k the

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integrate constant; σ and $\dot{\varepsilon}$ are stress and strain rate respectively. If substituting subscript x by l it denotes one-dimensional tension, by b it denotes two-dimensional bulge. For the case of one-dimensional tension, the parameters are shown in Fig. 1 (curve 1). For two-dimensional free bulge, σ and $\dot{\varepsilon}$ are equivalent stress and equivalent strain rate on the pole respectively, and the rest are shown in Fig. 1 (curve 2).

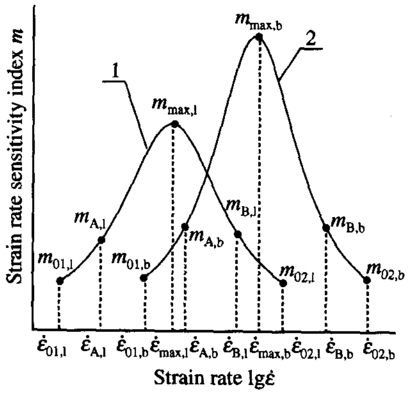

Fig. 1. The curve of $m-\lg\dot{\varepsilon}$.

If the curve of $m-\lg\dot{\varepsilon}$ is symmetric, there is

$$m_{A,x}=m_{B,x}=m_{k,x}=(m_{\max,x}+2m_{01,x})/3$$
$$=(m_{\max,x}+2m_{02,x})/3,$$

$$\eta_x=\lg\dot{\varepsilon}_{\max,x}-\lg\dot{\varepsilon}_{A,x}=\lg\dot{\varepsilon}_{B,x}-\lg\dot{\varepsilon}_{\max,x}.$$

If the curve of $m-\lg\dot{\varepsilon}$ is unsymmetrical, there is

$$m_{A,x}\neq m_{B,x},\quad \eta_{a,x}\neq\eta_{b,x},$$
$$m_{ka,x}=m_{A,x}=(m_{\max,x}+2m_{01,x})/3,$$
$$m_{kb,x}=m_{B,x}=(m_{\max,x}+2m_{02,x})/3,$$
$$\eta_{a,x}=\lg\dot{\varepsilon}_{\max,x}-\lg\dot{\varepsilon}_{A,x},$$
$$\eta_{b,x}=\lg\dot{\varepsilon}_{B,x}-\lg\dot{\varepsilon}_{\max,x}.$$

2 Solving the limiting radius of superplastic fill-forming bulge

According to Holt's method, Song et al. considered the stage of filling grooves as a plane strain process and assumed that the material obeyed the constant-volume law and the incremental theory of Hill[6] for an anistropic metal sheet. By a mechanical analysis, the expressions of filling-forming bulge are shown as follows:

$$\sigma=\frac{\sqrt{1+2R}}{1+R}qx^{1-a},\tag{3}$$

$$\dot{\varepsilon}=-\frac{1+R}{\sqrt{1+2R}}\frac{a}{x}\frac{\mathrm{d}x}{\mathrm{d}t}.\tag{4}$$

where the signification of parameters in Eqs. (3) and (4) is shown in Fig. 2, in which $x=\rho/\rho_0$ is the non-dimensional radius; ρ_0 and s_0 are the radius and thickness when the sample is touching grooves; ρ is the instantaneous radius in filling grooves; $q=\rho_0P/s_0$ the equivalent pressure;

$$a=(2\cot(\gamma/2)-(\pi-\gamma))/(\pi-\gamma),$$

γ the angle of groove; R the anisotropic coefficient in the thickness direction.

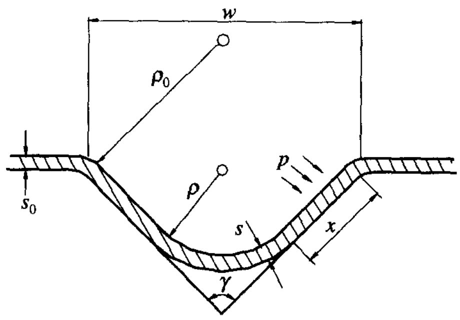

Fig. 2. The diagram of superplastic fill-forming bulge.

2.1 Solving the minimal radius of fill-forming bulge by uniaxial tensile constitutive equation with variable m

Substituting Eqs. (3) and (4) into the uniaxial tensile constitutive equation with variable m (Eq. (1)), one gets

$$-\frac{1+R}{\sqrt{1+2R}}\frac{a}{x_l}\frac{\mathrm{d}x_l}{\mathrm{d}t}=\dot{\varepsilon}_{\max,l}\exp\left\{\frac{\eta_l\ln10}{\sqrt{m_{\max,l}/m_{k,l}-1}}\cdot\tan\left(\frac{\sqrt{m_{\max,l}/m_{k,l}-1}}{m_{\max,l}\eta_l}\cdot\lg\left(\frac{\sqrt{1+2R}}{(1+R)k_l}qx_l^{1-a}\right)\right)\right\}.$$

Simplifying the above equation, one gets

$$\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{\dot{\varepsilon}_{\max,l}x_l\sqrt{1+2R}}{(1+R)a}\exp\left\{\frac{\eta_l\ln10}{\sqrt{m_{\max,l}/m_{k,l}-1}}\cdot\tan\left(\frac{\sqrt{m_{\max,l}/m_{k,l}-1}}{m_{\max,l}\eta_l}\cdot\lg\left(\frac{\sqrt{1+2R}}{(1+R)k_l}qx_l^{1-a}\right)\right)\right\}.\tag{5}$$

Substituting Eq. (5) into Eq. (4), the necessary strain rate $\dot{\varepsilon}_l$ corresponding to non-dimensional radius x_l can be obtained:

$$\dot{\varepsilon}_l = \dot{\varepsilon}_{\max,l}\exp\left\{\frac{\eta_l\ln 10}{\sqrt{m_{\max,l}/m_{k,l}-1}}\cdot\tan\left(\frac{\sqrt{m_{\max,l}/m_{k,l}-1}}{m_{\max,l}\eta_l}\cdot\lg\left(\frac{\sqrt{1+2R}}{(1+R)k_l}q_l x_l^{1-a}\right)\right)\right\}. \tag{6}$$

It is generally accepted that when the strain rate $\dot{\varepsilon}_B$ is several orders smaller than the optimum strain rate $\dot{\varepsilon}_{\max}$, the material loses its superplasticity and deformation stops. Therefore, the criterion to establish minimal filling radius under constant pressure is

$$\dot{\varepsilon}\,|_{x=x_{\min,l}} \to \dot{\varepsilon}_B. \tag{7}$$

Substituting Eq. (7) into Eq. (6), we have the relationship between the limiting radius and the equivalent pressure as

$$x_{\min,l}^{a-1} = D_l q_l, \tag{8}$$

where

$$D_l = (\sqrt{1+2R}/[(1+R)k_l]) \cdot 10^{-\frac{m_{\max,l}\eta_l}{\sqrt{m_{\max,l}/m_{k,l}-1}}\arctan\left[\frac{\sqrt{m_{\max,l}/m_{k,l}-1}\ln(\varepsilon_B/\varepsilon_{\max,l})}{\eta_l\ln 10}\right]}. \tag{9}$$

2.2 Solving the minimal radius of fill-forming bulge by bulging constitutive equation with variable m

Similar to the above methods used in Section 2.1, the criterion to analyze minimal fill-forming radius by the two-dimensional bulging constitutive equation with variable m is still Eq. (7). Substituting Eqs. (3) and (7) into the superplastic bulging constitutive equation with variable m (Eq. (1)), we have the relationship between the limiting radius and the equivalent pressure as

$$x_{\min,b}^{a-1} = D_b q_b, \tag{10}$$

where

$$D_b = (\sqrt{1+2R}/[(1+R)k_b]) \cdot 10^{-\frac{m_{\max,b}\eta_b}{\sqrt{m_{\max,b}/m_{k,b}-1}}\arctan\left[\frac{\sqrt{m_{\max,b}/m_{k,b}-1}\ln(\varepsilon_B/\varepsilon_{\max,b})}{\eta_b\ln 10}\right]}. \tag{11}$$

3 Comparison of theoretical and experimental results

3.1 Experiment

3.1.1 Tension All specimens were cut from Zn-wt22% Al superplastic alloy sheet with an original thickness (s_0) of 2 mm and its dimension within gauge length was 10 mm × 5 mm. The experimental temperature was 270 ℃ and was maintained constant for 8 min. The anistropic parameter R in the thickness direction is 0.58. The experimental curve of $m-\lg\dot{\varepsilon}$ corresponding to uniaxial tensile state is shown in Fig. 3, and the constitutive parameters of uniaxial tensile state corresponding to Eq. (1) are obtained:

$$\begin{cases} m_{\max,l} = 0.56, & \dot{\varepsilon}_{\max,l} = 4\times10^{-3}\,\mathrm{s}^{-1}, & m_{k,l} = 0.45, \\ \dot{\varepsilon}_{k,l} = 2\times10^{-2}\,\mathrm{s}^{-1}, & k_l = 14.11\,\mathrm{MPa}, & \eta_l = 0.7. \end{cases} \tag{12}$$

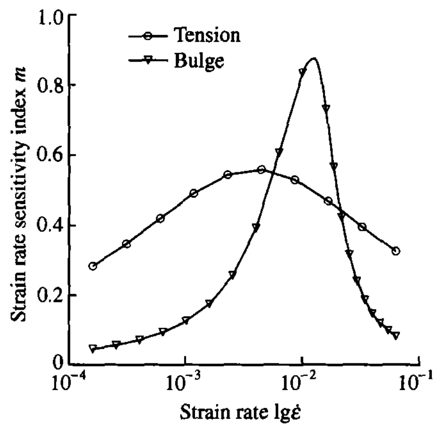

Fig. 3. The experimental curves of $m-\lg\dot{\varepsilon}$ by tensile and bulging test.

3.1.2 Free bulge The material and temperature are the same as above. The temperature of the specimen was maintained constant for 20 min. The radius of the specimen was 70 mm and the radius of pressing rim r_0 was 50 mm. Bulging the samples with the photoelectric measuring device described in Ref. [7], the experimental curve of $m-\lg\dot{\varepsilon}$ obtained by jump pressure method[8] is shown in Fig. 3, and when Eq. (1) is in two-dimensional free bulge state the corresponding constitutive parameters are

$$\begin{cases} m_{\max,b} = 0.875, & \dot{\varepsilon}_{\max,b} = 1.27\times10^{-2}\,\mathrm{s}^{-1}, & m_{kb,b} = 0.656, & \dot{\varepsilon}_{ka,b} = 0.7\times10^{-2}\,\mathrm{s}^{-1}, \\ \eta_{a,b} = 0.26, & \dot{\varepsilon}_{kb,b} = 1.7\times10^{-2}\,\mathrm{s}^{-1}, & \eta_{b,b} = 0.13, & k_b = 16.51\,\mathrm{MPa}. \end{cases} \tag{13}$$

3.1.3 Fill-forming bulge The specimens and experimental conditions were the same as those in the case of free bulge except for putting the fill-forming die of quartz glass into the clamping die. Photographs were taken for the forming sample at the focus of the lens, and measuring the limiting radius $\rho_{\min}$ by amplifying it tenfold. Three samples were measured under each pressure condition and the average values are shown in Fig. 4. Because the sheet would be bulged freely before it touched the cone wall, the fill-forming die could be devised at a free bulging height H_0 of 4 mm. Substituting r_0 and H_0 into $\rho_0 = (r_0^2 + H_0^2)/2H$[9], we will get the free bulging radius ρ_0 at 314.5 mm.

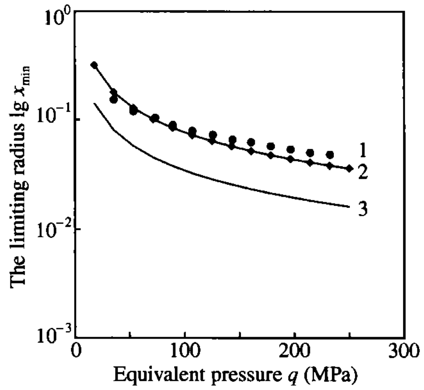

Fig. 4. The theoretical curves of $\lg x_{\min} \sim q$ and its experimental data. 1, Experimental data; 2, by two-dimensional bulging constitutive equation with variable m; 3, by one-dimensional tensile constitutive equation with variable m.

3.2 Theory

The experimental curve of $m - \lg\dot{\varepsilon}$ in bulging stress state is unsymmetrical, so the material will exert its superplastic potential only if the strain rate in fill-forming bulge follows the rule shown as the left curve in Fig. 3. Substituting

$\gamma = 90°$, $\rho_0 = 314.5\,\text{mm}$, $\lg(\dot{\varepsilon}_B/\dot{\varepsilon}_{\max}) = -3.7$ together with Eqs. (12), (13) into Eqs. (9), (11) respectively, one would get the material bulging limiting constants under constant pressure corresponding to one-dimensional tensile and two-dimensional free bulging constitutive equation with variable m

$$\begin{cases} D_l = 5.95253 \times 10^{-1}, \\ D_b = 2.25431 \times 10^{-1}. \end{cases} \tag{14}$$

Substituting Eq. (14) into Eqs. (8) and (10), we get the quantitative limiting equations between non-dimensional radius and equivalent pressure under constant pressure as

$$\begin{cases} x_{\min,l}^{-0.72676} = 5.95253 \times 10^{-1}\, q_l, \\ x_{\min,b}^{-0.72676} = 2.25431 \times 10^{-1}\, q_b. \end{cases} \tag{15}$$

The theoretical curves of limiting radius based on Eq. (15) are shown in Fig. 4.

3.3 Comparison of theoretical and experimental results

It is shown in Fig. 4 that although the theoretical curve and its change trend of fill-forming bulge by one-dimensional tensile constitutive equation with variable m is close to that by two-dimensional bulging constitutive equation, there still exists much discrepancy in quantity. The experimental data shown in Fig. 4 are in fair agreement with the theoretical curve by the two-dimensional bulging constitutive equation with variable m in most cases, but there still exists deviation, which might arise partly from the use of complete sticking friction between the material and the die wall, partly from both assumptions of uniform thickness and plane-strain. In fact, there exists gliding friction between the die and the specimen; and the part without touching the die is in non-uniform thickness and in plane-stress state. Therefore, it is necessary to correct the theoretical results.

Eq. (10) can be changed into

$$\left.\begin{aligned} x_{\min,b}^{a-1} &= A \\ A &= D_b q_b \end{aligned}\right\}. \tag{16}$$

Eq. (16) can be divided into two parts shown in Fig. 5. The left part is a family of $A \sim x_{\min,b}$ curve corresponding to different γ, which is only relevant to the angle of groove γ, not relevant to the character of material and its original dimensions. The right part $A \sim q_b$ is a universal curve independent of γ, and its slope D_b is only relevant to constitutive parameters in constitutive equation. Once the constitutive equation is determined, $A \sim q_b$ is accordingly determined. The discrepancy between theoretical curve and experimental data is only represented in the part of $A \sim \lg x_{\min,b}$, and the discrepancy value is getting smaller with the smaller $x_{\min,b}$. So, it is supposed that $f(x)$ is a correcting function by which the theoretical curve can be corrected to equal the experimental one. Simulating the experimental data in Fig. 5 by a computer, the fitting curve is

$$A = 0.413135 x_{\min,b}^{0.960559}. \tag{17}$$

According to the second expression in Eqs. (15) and (17), there is

$$f(x) x_{\min,b}^{-0.72676} = 0.413135 x_{\min,b}^{0.960559}. \tag{18}$$

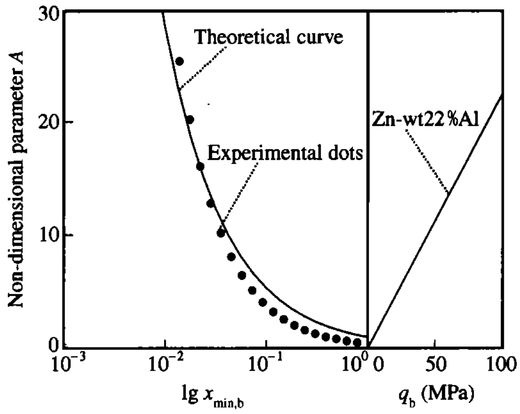

Fig. 5. The theoretical curve of limiting radius of fill-forming bulge by bulging constitutive equation with variable m and its experimental data at $\gamma = 90°$.

Simplifying Eq. (18) yields

$$f(x) = 0.413135x_{\min,b}^{0.233799}. \tag{19}$$

When the angle of groove is $\gamma = 90°$, Eq. (19) fits for any materials and any original dimensions.

4 Conclusions

(i) The mechanical problems of superplastic fill-forming bulge cannot be dealt with one-dimensional tensile constitutive equation with variable m, but superplastic free bulging constitutive equation with variable m that based on a two-dimensional bulging stress state can truly reflect deformation rules of fill-forming bulge.

(ii) From the analyzing process of correcting function $f(x)$, it can be concluded that when the angle of groove γ is determined, $f(x)$ is not relevant to material's character but only relevant to its deformation process, so it adapts to any materials and any original dimensions.

(iii) There exists a deviation between theoretical and experimental results when applying two-dimensional bulging constitutive equation. The primary reason is that we suppose the specimen is in uniform deformation and it ceases to deform once it touches the die. This issue will be discussed specially in another paper.

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